The complete relativistic kinetic model of violation of symmetry in isotopic expanding plasma and production of baryons in hot Universe. I. Exact model.

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Abstract

The complete model of production of baryons in expanding primordial symmetrical hot Universe is constructed in terms of general relativistic kinetic theory.

1 Introduction

An important example of local thermodynamical equilibrium (LTE) breaking by massive particles during the cosmological expanding is process of generation of baryons (baryogenesis) in primordial baryon - symmetrical Universe. On [1], [2] there was promoted a hypothesis that observing baryon skewness of Universe

$$\delta_b = \frac{n_B}{n_\gamma} \sim 10^{-9} \qquad (1)$$

is caused by CP- non - invariant processes, which break the conservation of baryon's charge. An example of such process is decay of superheavy X- bosons, to which Grand Unified Theories (GUT's) adduce:[1]

$$X \rightleftarrows \bar{q} + \bar{q}; \ \ X \leftrightarrows q + l, \qquad (2)$$

where q - quark's symbol, l - lepton's symbol. In [3], however is shown that if given processes are run in LTE's conditions then even the existence of - non-invariance and the breaking of baryon charge's conservation is insufficiently for appearance of baryon's excess over anti-baryons. For production of the baryon's charge in primordial baryon-symmetrical Universe besides stated facts require the breaking of LTE in reactions of such type (2) and withdrawal of baryons from statical equilibrium's conditions. Such possibility:

$$\tau_x \gtrsim t \qquad (3)$$

(τ_x - time of half-decay of X - boson, t - cosmological time) is realized under condition [4]:

$$m_X > \alpha_X m_{pl} \sqrt{N}, \qquad (4)$$

(m_X - mass of X-boson, $m_{pl} = \sqrt{\frac{\hbar c}{G}}$ - Planck's mass, N - number of degrees of freedom). Condition (4) stringently limits from below allowable values of X-bosons masses.

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[1] Here and further $\bar{a}$ means antiparticle a.

In Ref. [4], [6] (see too [7], [8] and etc.) there were made estimates of baryon skewness of Universe with the assumption of LTE's breaking. For attitude of baryons number's density to density of total entropy, $\mathcal{S}$, there was obtained:

$$\delta_{\mathcal{S}} = \frac{n_b}{S} \simeq \frac{45\zeta(3)}{4\pi^4} \frac{N_X}{N} \Delta r, \tag{5}$$

where $\zeta(x)$ - ζ - Rieman's function, N_X - the number of types of superheavy X-bosons, Δr - difference of comparative probabilities of decays in channels

$$X \to q + l \ \ \bar{X} \to \bar{q} + \bar{l},$$

which appears in consequence of breaking of CP - invariance. Further in papers [9]-[11] on basis of relativistic kinetic theory there were made numerical accounts of attitude n_B/S, which basically confirmed foregoing estimates. In cited earlier papers on the basis of made calculations there was found the inferior limit of X - boson's mass:

$$m_X \geq 10^{16} \text{Gev}, \tag{6}$$

which created considerable difficulties for standard $SU(5)$ theory.

Previously mentioned papers however have considerable deficiency - instead of direct solution of kinetic equations for X - bosons under conditions of *considerable* LTE's breaking boson functions of distribution are described by quasi-equilibrium (Bolzman's) distributions, which parameters are defined from hydrodynamical equations for moments of distribution's function, i.e., in this papers actually is used hydrodynamical method of Grad. According to results of relativistic kinetic theory (see for example, [12]-[14]), global thermodynamical equilibrium in homogeneous isotropic expanding plasma is reached only in extreme non-relativistic limit, or in extreme ultrarelativistic limit. In range of intermediate energies of particles and on conditions that LTE is breaking, the distribution of particles is not approximated by equilibrium distribution. In Ref. [17]-[18] of one of authors in the framework of kinetic theory there was found non-equilibrium function of distribution of X - bosons and shown that this function may essentially differs from equilibrium function. Therefore results obtained in [9]-[11]are correct if *strong* inequality (4) is applied, but needed a correction in other areas. The estimate (5) although seems to appear sufficiently accurate, but doesn't describe such situations, when condition (4) is not realized while width of covering of experimental and theoretical implications δ_S permits possibility when baryon charge is produce in conditions, that are less favorable than (4).

The answer to the question, what kind will be magnitude δ_S at $m_X \leq \alpha_X m_{pl}\sqrt{N}$, could be given only by detailed kinetic analysis. On the assumption of confidence to the hypothesis of primordial baryon- and charge- symmetrical Universe, such analysis from the other side allows more definitely indicate range of possible implications of fundamental constants of Grand Unified Theories. In the middle of 80-s - beginning of 90-s by one of authors was formulated the kinetic model of describing of symmetry's breaking processes and there were

obtained some estimations, following from this model (Ref. [17]-[19]). Specifically on basis of obtained estimations there was formulated a hypothesis that accounting of kinetic of baryogenesis process will permit to reduce the low estimation of masses of superheavy bosons to quantity of order $5 \cdot 10^{14}$Gev. However there wasn't executed detailed analysis of this model in this papers and on account of external, researches of this problem weren't complete. The purpose of this paper is exactly carrying out of more detailed analysis of the kinetic model of baryogenesis and construction of the numerous model of considered events. Let us notice that though this paper contains results for concrete model of interactions based on the minimal $SU(5)$ symmetry, the generalization of these results for other analogical field theories isn't hard - it is reduce to arithmetical recalculation of corresponding coefficients.

2 The algebra of interactions

Let us consider for example standard $SU(5)$ model of interactions (see for example [8]). There are involve 12 vectorial calibrating bosons which represent themselves 2 charged color triplets in this model:

$$\{X^i_{v,-4/3}, \ \bar{X}^i_{v,4/3}; \ X^i_{v,-1/3}, \bar{X}^i_{v,1/3}\}$$

- i - color index (red, green, blue), the subscript - electrical charge - and 12 scalar Higgs's bosons:

$$\{X^i_{s,-1/3}, \ \bar{X}^i_{s,1/3}; \ X^i_{s,-4/3}, \bar{X}^i_{s,4/3}\}.$$

In more common models the number of vectorial bosons reduces to 24, - and 2 more triplets add at the same time

$$\{X^i_{v',-1/3}, \ \bar{X}^i_{v',1/3}; \ X^i_{v',2/3}, \bar{X}^i_{v',-2/3}\},$$

and the number of Higgs's bosons reduces to 30, and at that add 3 $SU(3)$ - triplets, which are included in 3 $SU(2)$ triplets:

$$\{X^i_{s',-1/3}, \ \bar{X}^i_{s',1/3}; \ X^i_{s',2/3}, \bar{X}^i_{s'-2/3}; \ X^i_{s',-4/3}, \ \bar{X}^i_{s',4/3}\}.$$

Superheavy X-bosons are often calling X, Y, Z-bosons according to their charges: $-4/3, -1/3, 2/3$. The electric charge's conservation laws allow to run only these reactions of decay/creation of these bosons:

$$X_{-4/3} \rightleftarrows \bar{q}_\alpha + \bar{q}_\alpha; \ X_{-4/3} \rightleftarrows q_\kappa + l_e;$$

$$X_{-1/3} \rightleftarrows \bar{q}_\alpha + \bar{q}_\kappa; \ X_{-1/3} \rightleftarrows q_\kappa + l_\nu; \ X_{-1/3} \rightleftarrows q_\alpha + l_e; \qquad (7)$$

$$X_{2/3} \rightleftarrows \bar{q}_\kappa + \bar{q}_\kappa; \ X_{2/3} \rightleftarrows q_\alpha + l_\nu$$

where α - index of apoquark ($\alpha = u, c, t$ - quarks with charge 2/3), κ - katoquark ($\kappa = d, s, b$ quarks with charge $-1/3$), e - symbol of charged lepton ($e = e, \mu, \tau_-$,

charge=-1), ν - symbol of neutral lepton ($\nu = \nu_e, \nu_\mu, \nu_\tau$). In reactions (7) color charge is also conserving so therefore in reactions of decay of X-boson and in reaction of quark annihilation antiquarks of different colors, additional to X-boson's color, take part in. For example:

$$\bar{X}^R_{-4/3} = d^R_r + e_- = \bar{u}^{\bar{G}}_l + \bar{u}^{\bar{B}}_l;$$

$$\bar{X}^G_{-4/3} = d^G_l + e_- = \bar{u}^{\bar{R}}_l + \bar{u}^{\bar{B}}_l,$$

where R, G, B - indications of red, green, blue colors; corresponding overlined indexes $\bar{R}, \bar{G}, \bar{B}$ - anticolors, which are equal to sums of corresponding additionals colors, l, r - indications of left and right particles. We may suppose, as it often doing that in right parts of reactions (7) particles only with the same charm take part in. In that case in standard variant $SU(5)$ there will be 90 couples of reactions of type (7), and in expended variant of theory - 198 couples[2]. Then, the number of degrees of freedom in standard variant of theory is equal:

$$N = \sum_B (2s+1) + \frac{7}{8} \sum_F (2s+1) = 185$$

- 24 vectorial bosons ($s = 1$), - 29 scalar bosons ($s = 0$), 36 quarks ($s = 1/2$), 12 leptons ($s = 1/2$). In expanded variants $SU(5)$ this number is even greater.

3 Kinetic equations

Let us for convenience express averaged on spin states invariant elements of transition matrix of decays of superheavy bosons with the help of nondimensional numbers ($r_i, \bar{r}_i$):

$$|M_{X \to ql_e}|^2 = \tfrac{1}{3}s^2 r_e; \qquad |M_{X \to ql_\nu}|^2 = \tfrac{1}{3}s^2 r_\nu;$$

$$\left|M_{\bar{X} \to \bar{q}\bar{l}_e}\right|^2 = \tfrac{1}{3}s^2 \bar{r}_e; \qquad \left|M_{\bar{X} \to \bar{q}\bar{l}_\nu}\right|^2 = \tfrac{1}{3}s^2 \bar{r}_\nu; \qquad (8)$$

$$|M_{X \to \bar{q}\bar{q}}|^2 = \tfrac{1}{6}s^2(1-r); \quad \left|M_{\bar{X} \to qq}\right|^2 = \tfrac{1}{6}s^2(1-\bar{r}),$$

($r = r_e + r_\nu, \bar{r} = \bar{r}_e + \bar{r}_\nu$), which should be also provide by indexes of electrical and color charges as well as by charms of quarks and leptons.

Through CPT - invariance averaged matric elements of reverse transition (annihilation) are equal:

$$|M_{ql_e \to X}|^2 = \tfrac{1}{3}s^2 \bar{r}_e; \qquad |M_{ql_\nu \to X}|^2 = \tfrac{1}{3}s^2 \bar{r}_\nu;$$

$$\left|M_{\bar{q}\bar{l}_e \to \bar{X}}\right|^2 = \tfrac{1}{3}s^2 r_e; \qquad \left|M_{\bar{q}\bar{l}_\nu \to \bar{X}}\right|^2 = \tfrac{1}{3}s^2 r_\nu; \qquad (9)$$

$$|M_{\bar{q}\bar{q} \to X}|^2 = \tfrac{1}{6}s^2(1-\bar{r}); \quad \left|M_{qq \to \bar{X}}\right|^2 = \tfrac{1}{6}s^2(1-r).$$

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[2]With account of decay and creating of superheavy antibosons.

Invariant matric elements of double-particle decay's probability are constant. Calculation of total probability of decay in one-loop approximation in the context of, for example, standard $SU(5)$ model gives:

$$W_{X \to ql} = \frac{\pi \alpha}{108}(2\pi)^4 m_X^2 \delta^{(4)}(P_F - P_I),$$

where P-total 4-momentums of initial and final conditions $\alpha \approx 1/45$. In that case:

$$\sum_{A,C} s^2 = \frac{8\pi m_X^2}{27},\tag{10}$$

where summation is carries over all charms and colors of particles participating in decay of given type of X -boson.

In further however we shall not define concretely s^2, supposing only:

$$s^2 = \mathrm{Const}.\tag{11}$$

Besides first order reactions (7) reactions of higher order, running with participation of superheavy bosons and breaking CP-invariance and also conservation laws of baryon and lepton charges also are possible. For example

$$qq' \xrightarrow{X} \bar{q}l; \ q\bar{l} \xrightarrow{X} \bar{q}\bar{q}'.\tag{12}$$

Let us consider further metric of homogenous isotropic Universe [3]:

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2).\tag{13}$$

Let further p^i - 4-momentum of a particle, corresponding to relation of normalization:

$$(p,p) = (p^4)^2 - p^2 = m^2 \Rightarrow E = \sqrt{m^2 + p^2},\tag{14}$$

where $p^2 = a^2(t)((p^1)^2 + (p^2)^2 + (p^3)^2)$ - square of kinematic momentum of particle, $E = p^4$ - kinetic energy of particles.

In Ref. [15] was shown that relativistic kinetic equations are asymptotically conformally invariant in ultrarelativistic limit:

$$\frac{< p^2 >}{m^2} \to \infty\tag{15}$$

to within $O(m^2/p^2)$. Thus in ultrarelativistic limit (15) when energies of all particles participating in reactions lot more than their rest masses, kinetic equations in Friedman's metric after corresponding conformal conversion coincide with kinetic equations in Minkovsky space, therefore homogenous isotropic expanding of ultrarelativistic plasma doesn't lead out from condition of *global* thermodynamical equilibrium. Further in opposite non-relativistic limit:

$$\frac{< p^2 >}{m^2} \to 0,\tag{16}$$

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[3] Here and further we choose system of units $\hbar = c = G = 1$.

when kinetic energies of all particles participating in process far less than their rest masses, global thermodynamical equilibrium is again restores in plasma. [16],[12].

Let's introduce according to [15] conformal momentums of particles $\mathbb{P}, \mathbb{Q}$, which are integrals of motion relative to metric (13) according to formula:

$$\mathbb{P} = a(t)p \qquad (= \sqrt{(p_1)^2 + (p_2)^2 + (p_3)^2}), \qquad (17)$$

conformal kinetic energy of particles $\mathbb{E} = a(t)E$:

$$\mathbb{E} = \sqrt{a^2(t)m^2 + \mathbb{P}^2} \qquad (18)$$

and corresponding *macroscopic conformal parameters of plasma* , which conserve constant at the ultrarelativistic stage of expanding:

$$\mathcal{T} = a(t)T - \qquad\qquad\qquad \text{temperature;} \qquad (19)$$
$$\mathcal{N} = a^3(t)n - \qquad\qquad \text{number density of particles;} \qquad (20)$$
$$\mathcal{E} = a^4(t)\varepsilon - \qquad\qquad\qquad \text{energy density.} \qquad (21)$$

Kinetic equations for plasma in expanding homogenous Universe with participation of X -bosons we will write in symbolic form [19]:

$$\frac{1}{a}\sqrt{a^2 m_X^2 + \mathbb{P}^2}\frac{\partial f_X}{\partial t} = \sum I_{X \to \bar{q}\bar{q}'} + \sum I_{X \to ql}; \qquad (22)$$

$$\frac{1}{a}\sqrt{a^2 m_X^2 + \mathbb{Q}^2}\frac{\partial f_F}{\partial t} = \sum I_{X \to FF'} + \sum^{(2)} I_{FF' \to ...}$$
$$+(\text{other interactions}), \qquad (23)$$

where $f_a(P,t)$ - phase-space densities, F - index of fermions, $\mathbb{P}$, $\mathbb{Q}$ - conformal momentums of X -bosons and fermions, correspondingly. Expression "other interactions" in (23) means integral of collisions for all other interactions which don't violate conservation of baryon and lepton charges: quark-gluon, lepton and quark decays of W- X-bosons, annihilation etc. All these reactions run in conditions of LTE and lead to establishment of thermal equilibrium of quarks and leptons. Integrals of interactions of particles we will write in relativistic-invariant form [20]:

$$I_a(x, P_a) =$$
$$-\sum_{f,i} \int \delta^{(4)}(P_f - P_i)(Z_{fi}W_{if} - Z_{if}W_{if})\underset{f,i}{\Pi'}d\pi, \qquad (24)$$

where summation is carrying out by all initial, i, and final, f, conditions of particles, integration $(d\pi')$ is carrying out by all four-dimensional momentum spaces, except space of particle of a- sort, W_{fi} W_{if} - corresponding invariant

scattering matrixes which relate with invariant amplitudes of scattering M_{if} by formulas [21]:

$$W_{fi} = (2\pi)^4 |M_{fi}|^2 \cdot 2^{-(N_i + N_f)} - \tag{25}$$

- N_i N_f - numbers of particles in initial and final conditions. In formulas (40) introduced products of initial and final conditions:

$$Z_{fi} = \prod_f f_f \prod_i (1 \pm f_i);$$

$$Z_{if} = \prod_i f_i \prod_f (1 \pm f_f), \tag{26}$$

where symbol "−" relates to fermions, "+" - to bosons. Invariant elements of momentum space volumes are equal:

$$d\pi_a = \frac{\rho_a \sqrt{-g} d^4 P_a}{(2\pi\hbar)^3} \delta((P_a, P_a) - \frac{m_a^2 c^2}{2}),$$

where ρ_a - statistical degeneracy factor.

Further, since reactions (7), (12) run under very high temperatures $T \gtrsim 10^{14}$Gev, it is possible with very high accuracy grade to mean that all fermions are ultrarelativistic, therefore:

$$f_{F_a} = \left[\exp(-\lambda_a + \mathbb{Q}/\mathcal{T}) + 1 \right]^{-1}, \tag{27}$$

where conformal temperature $\mathcal{T} = aT$, is constant in ultrarelativistic equilibrium plasma, and *adduced* chemical potentials

$$\lambda_a = \frac{\mu_a}{T}$$

are satisfy to series of conditions of chemical equilibrium, corresponding to reactions executing in conditions of LTE. Combining algebraic equations corresponding to these conditions we may come to such conclusion: chemical potentials of each type of quark aren't depend on their own color and charm. In that way it remains only 4 independent chemical potentials which for simplicity sake, following [8], we will denote by symbols of particles themselves - u, d, e, ν:

$$u = \lambda_{q_\alpha}; \;\; d = \lambda_{q_\kappa}; \;\; e = \lambda_{l_e}; \;\; \nu = \lambda_{l_\nu}, \tag{28}$$

for them at that as for ultrarelativistic particles in conditions of LTE applies known condition of antisymmetry:

$$\bar{\lambda}_a = -\lambda_a. \tag{29}$$

In situation when rest mass of neutral leptons is equal to zero, their chemical potential is also equal to zero: $\nu = 0$, as chemical potential of massless particles being in thermodynamical equilibrium [4]. In that case remain only three independent variables: u, d, e.

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[4]see for example, [22].

3.1 Kinematic equations for fermions

At completion of chemical equilibrium conditions the last component in right part (23) becomes a zero. The second component in right part (23) describes all reactions executing with violence of CP-invariance, in which quarks, leptons and virtual X-bosons participate. Since these integrals completely defined by equilibrium functions of distribution of quarks, leptons and other particles, then using *functional Bolzman's equations* [23]

$$Z_{fi} - Z_{if} = 0 \Rightarrow \sum_i \lambda - \sum_f \lambda' = 0, \tag{30}$$

Fermion integrals of interaction can be write down in the form of:

$$\sum I_{FF' \to \ldots} = -\sum \int d\pi_F d\pi_F d\pi_{F'} f_F f_{F'} \times$$
$$\times \int \prod d\pi_k (1 \pm f_k)(W_{FF' \to \ldots} - \bar{W}_{\bar{F}\bar{F}' \to \ldots}). \tag{31}$$

There is no need to concretize probabilities of many-particle conversions W $\bar{W}$ for calculation of these integrals - it is enough to use optical theorem which is consequence of unitarity of S-matrix, [8], [24]:

$$\sum_k \int \prod d\pi_k (1 \pm f_k)(W_{if} - W_{fi}) = 0, \tag{32}$$

where summation is carrying out by all *final* conditions of reactions $FF' \to \ldots$. So in consequence of optical theorem we will obtain equality:

$$\sideset{}{'}\sum_{X,F'} (1 + f_X)(W_{X \to \bar{F}\bar{F}} - W_{\bar{X} \to FF'}) d\pi_k +$$
$$\sideset{}{'}\sum_k (1 \pm f_k)(W_{FF' \to \ldots} - W_{\bar{F}\bar{F}' \to \ldots}) d\pi_k = 0. \tag{33}$$

$$\tag{34}$$

Using this result in equations (23) and integrating them by momentum space of fermions we will receive equations which in further will be used for definition of chemical potentials of fermions:

$$\frac{d\mathcal{N}_F}{dt} = a(t) \sideset{}{'}\sum_{\bar{X},F'} \int W_{X \to FF'} d\pi_X d\pi_F d\pi_{F'} \times$$
$$[f_X(1 - f_F)(1 - f_{F'}) - (1 + f_X)f_F f_{F'}]. \tag{35}$$

Last equations can be still simplified by accounting of correlations which are correct to equilibrium fermion distribution functions (27):

$$\int d\pi_F d\pi_{F'} \delta^{(4)}(p_X - p_F - p_{F'}) f_F f_{F'}$$

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$$= \frac{1}{2^3 \pi^5} f_X^0(F + F') \beta(F, F'); \tag{36}$$

$$\int d\pi_F d\pi_{F'} \delta^{(4)}(p_X - p_F - p_{F'})(1 - f_F)(1 - f_{F'})$$

$$= \frac{1}{2^3 \pi^5}[1 + f_X^0(F + F')]\beta(F, F'), \tag{37}$$

where incorporated notations [19]:

$$f_X^0(F + F') = [\exp(-\lambda_F - \lambda_{F'} + E_X/T) - 1]^{-1};$$

$$\beta(F, F') = \beta(F', F) = \beta(F) + \beta(F'); \tag{38}$$

$$\beta(F) = \frac{T}{p} \ln \frac{1 + \exp(-\lambda_F + p_+/T)}{1 + \exp(-\lambda_F + p_-/T)} - \frac{1}{2},$$

$$p_\pm = \frac{1}{2}(E \pm p); \quad E = E_X = \sqrt{m_X^2 + p^2}. \tag{39}$$

Function $\beta(F, F')$ is statistical factor which in Bolzman's statistics ($\lambda \to \infty$) is equal to 1. Arguments $\mathbb{P}$ and t of functions $f_X^0 \quad \beta$ are dropped for short. In necessary situations we will write explicitly:

$$f_X^0(x; \mathbb{P}, t); \qquad \beta(x, \mathbb{P}, t).$$

Subject to (36),(37) equations (35) can be essentially simplified:

$$\frac{d\mathcal{N}_F}{dt} = a(t) \sum_{X,F}{}' \frac{\rho_X}{(2\pi)^3} |M_{X \to FF'}|^2 \quad \times$$

$$\int_0^\infty \frac{p^2 [f_X - f_X^0(F + F')]\beta(F, F')dp}{\sqrt{m_X^2 + p^2}}. \tag{40}$$

Summation in (40) is carrying out by all X-bosons, moreover in quark-quark channels sum of colors of two quarks is complementary to color of X-boson, thus to each type of X-boson are correspond 2 addends in sum.

3.2 Kinetic equations for X-bosons

For X-bosons optical theorem assumes the form:

$$\sum_{F,F'} d\pi_F d\pi_{F'}(1 - f_F)(1 - f_{F'})(W_{x \to \bar{F}\bar{F}'} - W_{\bar{X} \to FF'}) = 0 \tag{41}$$

and thus boson equations can be wrote in the form:

$$\sqrt{a^2(t)m_X^2 + \mathbb{P}^2} \frac{\partial f_X}{\partial t} =$$

$$-\frac{a(t)}{4\pi} \sum_{F,F'} |M_{X \to \bar{F}\bar{F}'}|^2 \beta(F, F')[f_X - f_X^0(F + F')]. \tag{42}$$

Let's draw attention to that important and *strict* fact, that kinematic equations for X-bosons become linear differential equations. Equations for antiparticles receive from (40) (42) by inversion of overlined indexes of particles - antiparticles.

If assume that spontaneous violence of $SU(5)$-symmetry happened at very early stages of cosmological evolution when X-bosons were still ultrarelativistic, then mentioned system of kinetic equations (40), (42) is necessary to solve with initial conditions corresponding to initial global thermodynamic equilibrium and initial baryon and lepton symmetry:

$$\lambda_a|_{t=0} = 0; \qquad f_X(\mathbb{P}, 0) = f_0^0(0) = [\exp(\mathbb{P}/\mathcal{T}_0) - 1]^{-1}, \tag{43}$$

where:

$$\mathcal{T}_0 = \mathcal{T}(0) = \left(\frac{45}{16\pi^3 N}\right)^{1/4}. \tag{44}$$

From (42) it is obvious that boson functions can be found in quadratures [14], [18]. These equations in standard mathematical notation look like:

$$\dot{f}_i + f_i A_i(t) = Y_i(t), \tag{45}$$

point here and further means time derivative, i, k - now indexes of X-bosons, and vectors A Y define by correspondences:

$$A_i(\mathbb{P}, t) = \frac{a(t)}{4\pi\sqrt{m_i^2 a^2(t) + \mathbb{P}_i^2}} \sum_k |M_{X_i \to F, F'}|^2 \beta(F, F'); \tag{46}$$

$$Y_i(\mathbb{P}, t) = \frac{a(t)}{4\pi\sqrt{m_i^2 a^2(t) + \mathbb{P}_i^2}} \times$$
$$\sum_k |M_{X_i \to F, F'}|^2 \beta(F, F') f_X^0(F + F'), \tag{47}$$

- summation in (46),(47) is carrying out by all channels of reactions over which given X-boson decays.

Solving equations (45) with initial conditions (43), (44), we will receive solutions for boson functions in quadratures:

$$f_X(\mathbb{P}, t) = f_X(\mathbb{P}, 0) \exp\left(-\int_0^t A_i(\mathbb{P}, t')dt'\right) +$$
$$\exp\left(-\int_0^t A_i(\mathbb{P}, t')dt'\right) \int_0^t \exp\left(\int_0^{t'} A_i(\mathbb{P}, t'')dt''\right) Y_i(\mathbb{P}, t'')dt''. \tag{48}$$

Substitution of obtained solution for boson distribution functions in kinetic equations for fermions (40) leads to closed system of nonlinear integro-differential equations relative to chemical potentials.

3.3 Conservation laws

In complete kinetic model of Universe it is necessary to add to equations (40), (42) another equations which define evolution of temperature and scale factor. First of these equations receives by integration of kinetic equations (22), (23) with weight E_i and following summation [23]:

$$\dot{\mathcal{E}} = \dot{a} \sum_X m_X (\mathcal{K}_X + \mathcal{K}_{\bar{X}}), \tag{49}$$

where $\mathcal{E}$ - summary conformal energy density:

$$\mathcal{E} = \frac{\pi^2 N' \mathcal{T}^4}{30} + \sum_X \frac{\rho_X}{2\pi^2} \int\limits_0^\infty f_X \mathcal{P}^2 \sqrt{a^2 m_X^2 + \mathcal{P}^2} d\mathcal{P}, \tag{50}$$

$$\mathcal{K}_X = \frac{a m_X \rho_X}{2\pi^2} \int\limits_0^\infty \frac{f_X \mathcal{P}^2}{\sqrt{a^2 m_X^2 + \mathbb{P}^2}} d\mathbb{P}, \tag{51}$$

where N' - number of degrees of freedom of $SU(5)$-model without accounting of X-bosons:

$$N' = N - N_X,$$

i.e., in standard $SU(5)$-model $N' = 185 - 53 = 132$. And equation that defines evolution of scale factor is Einstein's equation in which we can put $k = 0$, taking into account that we consider very early Universe $t \to 0$:

$$\dot{a}^2 = \frac{8\pi}{3} \mathcal{E}. \tag{52}$$

Equations (40), (42), (49), (52) jointly with definitions (50), (51) and initial conditions (43), (44) are the complete system of equations which describe dynamics of baryogenesis. Let us consider some *strict* implications of these equations.

1. Let us suppose that X-bosons lie in thermal equilibrium, i.e., ignoring kinetic equations for X-bosons (42), we will put in (40)

$$f_X = f_X^0(\lambda_X) = \left[\exp\left(-\lambda_X + \frac{\sqrt{a^2 m_X^2 + \mathbb{P}^2}}{\mathcal{T}} \right) - 1 \right]^{-1},$$

where $\lambda_X = -\lambda - \lambda'$. Then exact solution of equations (40), satisfying to initial conditions (43) will be:

$$\mathcal{N}_a = \mathrm{C}onst; \qquad \lambda_a = 0, \tag{53}$$

- i.e., in conditions of statistical equilibrium of X-bosons baryogenesis doesn't execute.

2. Let us suppose that interactions CP are invariant, i.e.:

$$W_{X \to \bar{F}, \bar{F}'} = W_{\bar{X} \to FF'}.$$

In this situation from (40), (42) follows again (53). Thus and in situation of CP-invariance of interactions baryogenesis doesn't execute.

3. Integrating kinetic equations for X-bosons (42) over momentum space and combining this result with equations (40) taking into account initial conditions (43) we will receive two *strict* implications:

$$\sum_{A,C} \left(-\frac{4}{3}\Delta n_{-4/3} - \frac{1}{3}\Delta n_{-1/3} + \right.$$

$$\left. +\frac{2}{3}\Delta n_{2/3} + \frac{2}{3}\Delta n_\alpha - \frac{1}{3}\Delta n_\kappa - \Delta n_e \right) = 0 \tag{54}$$

- law of conservation of electric charge ($\sum e_a n_a = 0$)

$$\sum_{A,C} \left[-\frac{2}{3}\left(\Delta n_{-4/3} + \Delta n_{-1/3} + \Delta n_{2/3} \right) + \right.$$

$$\left. +\frac{1}{3}\left(\Delta n_\alpha + \Delta n_\kappa \right) - \Delta n_e - \Delta n_\nu \right] = 0 \tag{55}$$

- law of conservation of difference of baryon and lepton charges where incorporated notation:

$$\Delta n_a = n_a - \bar{n}_a. \tag{56}$$

Summation in (54), (55) is carrying out by all charms and colors of particles.

4. Supposing now that all X-bosons decayed i.e., proceeding to limit $t \to \infty$ and combining correspondences (54), (55) with accounting of conditions of symmetry (29) and notations (28), we will receive:

$$u(\infty) = 2d(\infty); \ u(\infty) = 2e(\infty) \Rightarrow d(\infty) = e(\infty). \tag{57}$$

From (57) follows that on completion of decay of X-bosons for each excess charged lepton it will corresponds one excess katoquark and two excess anoquark, what subject to colors will give one excess achromic baryon to one lepton. Hence final baryon charge of Universe will be equal to its lepton charge:

$$\Delta n_b(\infty) = \Delta n_e(\infty). \tag{58}$$

Thus for finding baryon asymmetry of Universe it is enough to define only one from three values:

$$u(\infty), \quad e(\infty), \quad d(\infty).$$

5. In simplest model of baryogenesis when CP-invariance violets only in decay channels of one type of X-bosons, $X_{-4/3}$, from (54), (55) strictly follows

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correspondence in all time of process:

$$e(t) = d(t), \tag{59}$$

and remain only two independent values $u(t)$ $d(t)$.

Concrete numerical model of baryogenesis will be published in the next article.

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References

[1] Sakharov A.D. The violation of CP-invariance, C-assymetry and baryon assymetry of Universe. *Letters in ZETP*,1967, vol. 5, p. 32-35.

[2] Kuzmin V.A. CP-non-invariance and baryon assymetry of Universe. *Letters in ZETP*, 1970, vol. 12, p. 335-337.

[3] Okun L.B., Zeldovich Ja.B. *Comments on Nucl. and Part. Physics*, 1976, V. 6, p. 69-73

[4] Ignat'ev A.Yu., Krasnikov N.V., Kusmin V.F., Tavhelidze A.N. *Phys. Lett.*, 1976, V.76B, p. 436-439

[5] Yoshimura H. Unified Gauge Theories and Baryon Number of the Universe. *Phys. Rev. Lett.*, 1978, V. 41, p. 281-284

[6] Weinberg S. Cosmological Production of Baryons. *Phys. Rev. Lett.*, 1979, V. 42, p. 850-853

[7] Weinberg S. Beyond the first Three Minutes. *Physics Scripta*, 1981, V. 21, p. 773-791

[8] Dolgov A.D., Zeldovich Ja.B. Cosmology and elementary particles. *Uspekhi fiz. nauk*, 1980, vol. 130, p. 559-614

[9] Fry J.M., Olive K.A., Turner M.S. Evolution of cosmological Baryon asymmetric. I. The role of gauge bosons. *Phys. Rev.D*, 1980, V. 22, p. 2953-2976

[10] Fry J.M., Olive K.A., Turner M.S. Evolution of cosmological Baryon asymmetric. II. The role of Higgs bosons. *Phys. Rev.D*, 1980, V. 22, p. 2977-2988

[11] Fry J.M., Olive K.A., Turner M.S. Hierarchy of cosmological Daryons Generation. *Phys. Rev. Lett*, 1980, V. 45, p. 2074-2077

[12] Ignat'ev Yu.G. About kinetical equation in General Relativity Theory. *Izvestiya Vuzov, Fizika*, 1979, vol.22, No 2, p. 72-76

[13] Ignat'ev Yu.G. Relativistic kinetics and cosmology. I. *Izvestiya Vuzov, Fizika*, 1980, vol.23, No 8, p. 42-47

[14] Ignat'ev Yu.G. Relativistic kinetics and cosmology. II. *Izvestiya Vuzov, Fizika*, 1980, vol.23, No 9, p. 27-32

[15] Ignat'ev Yu.G. Relativistic kinetic theory and conformal transformations. *Izvestiya Vuzov, Fizika*, 1982, vol.25, No 4, p. 92-96

[16] V.B.Magalinsky. Kinetics of small pereturbations of space-homogenous gravitating medium. *Astronom. Journ.*, 1972, vol. 49, No. 5, p. 1017-1025

[17] Ignat'ev Yu.G. Relativistic kinetics of baryogenesis in hot Universe. *Actual theoretical and experimantal problems of relativity theory and gravitation.* Report of Soviet conference, Moscow, 1984, p. 19-21

[18] Ignat'ev Yu.G. Relativistic kinetics of baryogenesis in hot Universe. *Astronom. Journ*, 1985, vol. 62, p. 633-638

[19] Ignat'ev Yu.G. Kinetic model of baryogenesis in symmetrical hot universe. In *Classical and Quantum-statistical problems of relativistic theory of gravitation*, Kazan, 1991, Izdatelstvo KSPI, p. 6-21

[20] Ignat'ev Yu.G. Relativistic kinetic equations for non-elastic interacting particles in gravitation field. *Izvestiya Vuzov, Fizika*, 1983, vol.26, No 8, p. 19-23

[21] Berestetsky V.B., Lifshitz E.M., Pitaevsky L.P. Relativistic quantum theory, Part I.// Moscow, Nauka, 1968

[22] Landau L.D., Lifshitz E.M. Statistic physics, Moscow, Nauka, 1964.

[23] Ignat'ev Yu.G. Conservation laws and thermodynamic equilibrium in general-relativistic kinetic theory of non-elastic interacting particles.//*Izvestiya Vuzov, Fizika*, 1983, vol.26, No 12, p. 2-14

[24] Pilkuhn H.M. Relativistic Particle Physics.// *Springer-Verlag*, New York-Heidelberg-Berlin, 1982